Using Proportional Relationships to Solve Ratio and Percent Problems

1 **GETTING THE IDEA**

Ratios and proportions can be used to solve many real-world problems, including **percent** problems. A percent is a ratio that compares a number to 100. For example, 20% can be represented as $\frac{20}{100}$.

There are many everyday examples of percent problems. Percents are used to calculate the total cost of a restaurant meal, including sales tax. Sales tax is a percentage of the total cost of the meal. In a restaurant, a customer also pays a tip, or gratuity, to the server. The tip is also a percentage of the total amount of the bill.

A salesperson's earnings may be calculated by adding the person's base pay or salary to a **commission**. The commission is based on a percent of the total amount of sales.

Since percents are ratios, proportions can be used to solve real-world percent problems.

Example 1

The sales tax on a coat that is priced at $45 is 8%. Find the total cost of the coat.

Strategy Set up and solve a proportion. Then add the amount of tax to the cost of the item.

Step 1 Write the percent as a ratio.

$$8\% = \frac{8}{100}$$

Step 2 Write a ratio comparing the amount of the tax to the price of the coat.

The amount of the tax is unknown. The price of the coat is $45.

$$\frac{x}{45}$$

Step 3 Set the two ratios equal and solve for x.

$$\frac{8}{100} = \frac{x}{45}$$

$$8 \cdot 45 = 100 \cdot x$$

$$360 = 100x$$

$$3.60 = x$$

So, the amount of the tax is $3.60.

Step 4   Add the amount of the tax to the cost of the coat.

$$\$45.00 + \$3.60 = \$48.60$$

Solution   The total cost of the coat is $48.60.

Example 2

Nancy and two friends paid a 15% tip for a restaurant bill of $53.74. Then they split the total evenly. How much did each person pay?

Strategy   **Set up and solve a proportion. Then add the amount of the tip to the bill and divide.**

Step 1   Set up a proportion to find the amount of the tip.

Write a ratio for the percent: $15\% = \dfrac{15}{100}$

Write a ratio comparing the unknown amount of the tip to the cost of the bill: $\dfrac{x}{53.74}$

Set the ratios equal: $\dfrac{15}{100} = \dfrac{x}{53.74}$

Step 2   Solve the proportion to find the amount of the tip.

$$\dfrac{15}{100} = \dfrac{x}{53.74}$$

$$15 \cdot 53.74 = 100 \cdot x$$

$$806.1 = 100x$$

$$8.061 = x$$

Round the amount to the nearest cent. The tip was $8.06.

Step 3   Find the amount each person paid.

Add to find the total of the bill plus the tip.

$$53.74 + 8.06 = 61.80$$

There are 3 people—Nancy plus 2 friends. So, divide the total by 3.

$$61.80 \div 3 = 20.60$$

Solution   Each person paid $20.60.

A **percent change** is the ratio of the change in a quantity to the original quantity, expressed as a percent. If the quantity is increasing, this is called a **percent increase**. If the quantity is decreasing, it is a **percent decrease**. A discount or markdown is a type of percent decrease. A markup is a type of percent increase.

Example 3

A lamp that originally cost $180 was marked down to $150. Find the percent of the discount.

Strategy   Subtract to find the amount of the discount. Set up and solve a proportion to find the percent.

Step 1   Subtract to find the amount of the discount.

$$180 - 150 = 30$$

Step 2   Set up a proportion.

Write a ratio to show the unknown percent. Then write a second ratio comparing the amount of the discount to the original cost.

$$\frac{x}{100} = \frac{30}{180}$$

Step 3   Solve the proportion.

$$\frac{x}{100} = \frac{30}{180}$$
$$x \cdot 180 = 100 \cdot 30$$
$$180x = 3{,}000$$
$$x = 16\frac{2}{3} \approx 16.7\%$$

Solution   The percent of the discount was about 16.7%.

Interest is the amount earned when money is deposited in a savings account or other type of investment. Interest can also be the extra amount paid when money is borrowed.

The original amount invested or borrowed is called the **principal**. To compute **simple interest**, solve a proportion to find the amount of interest earned in 1 year. Then, multiply by the number of years. Alternatively, you can use the simple interest formula.

Simple Interest Formula

$$I = prt$$

where I is the amount of the interest, p is the principal, r is the interest rate expressed as a decimal, and t is the time in years

Example 4

Dante invests $3,500 and earns 2% in simple interest each year. Assuming he makes no additional deposits or withdrawals, how much will his investment be worth in 5 years?

Strategy   Find the interest earned in 1 year. Multiply by the number of years. Then, add this to the principal. Check your answer using the simple interest formula.

Step 1   Write and solve a proportion to find the interest for 1 year.

$$\frac{2}{100} = \frac{x}{3,500}$$

$$2 \cdot 3,500 = 100 \cdot x$$

$$7,000 = 100x$$

$$70 = x$$

In 1 year, the principal earns $70 in interest.

Step 2   Multiply by the number of years.

$$5 \cdot 70 = 350$$

Step 3   Add the total interest to the principal.

$$350 + 3,500 = 3,850$$

Step 4   Check your answer using the simple interest formula.

$$2\% = \frac{2}{100} = 0.02, \text{ so:}$$

$$I = prt = 3,500 \cdot 0.02 \cdot 5 = 70 \cdot 5 = 350$$

This is the same amount of interest found using proportions, so the sum will be the same, 3,850.

Solution   In 5 years, the investment will be worth $3,850.

The **percent error** shows how close an estimate or measurement is to the actual value. Percent error is often used when describing the results of a scientific experiment.

Example 5

John needs to measure his windows, so he can buy new curtains. He measured the height of his windows to be $4\frac{3}{4}$ feet. The actual height is 5 feet. Find the percent error in John's measurement.

Strategy Find the absolute value of the difference of the estimated and actual heights. Use the actual height to write a proportion. Solve to find the percent.

Step 1 Find the absolute value of the difference of John's measurement and the actual measurement.

$$|\text{John's measurement} - \text{actual measurement}| = \left|4\frac{3}{4} - 5\right| = \left|-\frac{1}{4}\right| = \frac{1}{4}$$

The difference is $\frac{1}{4}$ foot.

Step 2 Set up a proportion.

You are looking for the percent error, so the first ratio should represent the percent.

The second ratio will compare the difference of the measurements to the actual measurement.

$$\frac{x}{100} = \frac{\frac{1}{4}}{5}$$

Step 3 Solve the proportion.

$$\frac{x}{100} = \frac{\frac{1}{4}}{5}$$
$$x \cdot 5 = 100 \cdot \frac{1}{4}$$
$$5x = 25$$
$$x = 5$$

Solution The percent error in John's measurement was 5%.

The chart below shows population changes in two towns over a period of 10 years. Which town had the greater percent increase in its population?

Population Changes in Ten Years

	2000	2010
Riverton Population	38,500	41,200
Mountainside Population	14,700	16,500

Determine the amount of the increase for Riverton.

$41{,}200 - 38{,}500 = $ _____

Write a ratio for the percent increase:

$$\frac{\text{amount of increase}}{\text{original population}} = \frac{\boxed{}}{38{,}500}$$

Set that ratio equal to $\frac{r}{100}$. Then solve for r, the percent increase for Riverton.

$$\frac{\boxed{}}{38{,}500} = \frac{r}{100}$$

_____ $\cdot 100 = $ _____ $\cdot r$

_____ $= $ _____

_____ $\approx r$

So, the population of Riverton increased by approximately ____%.

Determine the amount of the increase for Mountainside.

$16{,}500 - $ _____ $= $ _____

Write a ratio for the percent increase:

$$\frac{\text{amount of increase}}{\text{original population}} = \frac{\boxed{}}{\boxed{}}$$

Set that ratio equal to $\frac{m}{100}$. Then solve for m, the percent increase for Mountainside.

$$\frac{\boxed{}}{\boxed{}} = \frac{m}{100}$$

_____ $\cdot 100 = $ _____ $\cdot m$
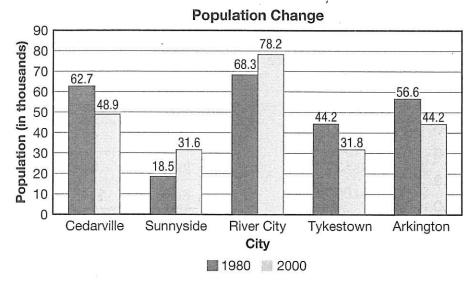
_____ $= $ _____

_____ $\approx m$

So, the population of Mountainside increased by approximately ____%.

The town of _____ had a greater percent increase in its population.

① The number of students enrolled in school increased by 12% from last year to this year. The enrollment last year was 15,500. What is the enrollment this year?

A. 1,860 students

B. 13,640 students

C. 15,686 students

D. 17,360 students

② Enrico earns a 3% commission on everything he sells at a store. On one day, he sold items priced at $86, $245, and $295. How much commission did he earn that day?

A. $18.78

B. $88.50

C. $187.80

D. $626.00

The graph below shows population changes in several cities. Use the graph for questions 3 and 4.

Population Change

③ Which city had a population decrease of over 25%?

A. Cedarville

B. Sunnyside

C. Tykestown

D. Arkington

④ Rounded to the nearest percent, what is the percent change in River City's population?

A. 10% increase

B. 13% increase

C. 14% increase

D. 15% increase

5. Four friends share equally the cost of a meal at a cafe. The cost of the food before sales tax and tip are included is $94. The friends want to pay a 15% tip based on this amount. They also need to pay a sales tax of 5% on the $94. How much does each person pay, including sales tax and tip?

A. $4.70

B. $14.10

C. $18.80

D. $28.20

6. At a furniture store, Tehmina paid $682.50, including sales tax, for a table priced at $650. How much will Tehmina pay, including sales tax, for a recliner priced at $740?

A. $32.50

B. $37.00

C. $772.50

D. $777.00

7. Hannah's band plays at events in their town and in other towns. Last year, they played at 18 events in their town and at 12 events in other towns. What percent of the events at which they played were in other towns?

A. 30%

B. $33\frac{1}{3}\%$

C. 40%

D. 60%

8. Carla estimated her town's population at 77,500. The actual population is 75,000. Which proportion can be used to find Carla's percent error?

A. $\frac{x}{100} = \frac{2,500}{75,000}$

B. $\frac{x}{100} = \frac{75,000}{77,500}$

C. $\frac{x}{100} = \frac{2,500}{77,000}$

D. $\frac{x}{100} = \frac{77,500}{75,000}$

9. The table below shows the original price and the sale price of some items at a store.

Item	Original Price	Sale Price
Jacket	$85.00	$74.79
Shoes	$24.95	$19.95
Jeans	$64.69	$55.00
Suitcase	$145.00	$124.99

For which item is the discount greater than 15%?

A. jacket

B. shoes

C. jeans

D. suitcase

10 Ahmal gets paid weekly at his summer job. He wants to save 15% of each paycheck. In four weeks, he earns these amounts: $212.50, $235.00, $215.75, and $199.87. What is the total amount he will save for these four weeks?

A. $31.88

B. $129.47

C. $733.65

D. $863.12

11 Half of what Trisha earned last summer came from painting houses and half from mowing lawns. She saved 18% of what she earned. If she saved $180, how much did she earn mowing lawns?

A. $500

B. $1,000

C. $1,620

D. $3,240

12 Maria earned $8 per hour as a camp counselor last summer. She gave 2% of her earnings to a charity. If she donated $40 to the charity, how many hours did she work?

A. 250 hours

B. 320 hours

C. 800 hours

D. 2,000 hours

13 From 1980 to 1981, a town's population decreased by 2%. From 1981 to 1982, the population increased by 5%. If the town's population was 6,200 in 1980, what was the population, rounded to the nearest whole number, in 1982?

A. 6,014

B. 6,076

C. 6,380

D. 6,386

14 A car's gas mileage is the number of miles it travels per gallon of gas. Janet's car used about 5 gallons of gas to drive 130 miles on the highway. She wants to estimate how much she will spend on gas for a 600-mile highway trip. How many dollars will she spend if gas costs $3.70 per gallon? Round your answer to the nearest cent.

A. $23.08

B. $26.00

C. $85.38

D. $481.00

15 Four people each invested $1,500 in an account that earns simple interest. Assuming no other deposits or withdrawals were made, which investment earned less than $100?

A. 2 years at 4.2%

B. 4 years at 2.8%

C. 6 years at 1.3%

D. 8 years at 0.75%

16 Frances measures the widths of two hairs. The actual width of each hair was 0.100 mm, and there was a 2% error in each measurement. Which of the following lengths could **not** be the sum of her two measurements?

A. 0.196 mm

B. 0.198 mm

C. 0.200 mm

D. 0.204 mm

17 Mario and Luis each earned $900 from summer internships after 10th grade. Mario put 80% of his earnings in a savings account that paid simple interest, and Luis invested 85% of his earnings in the stock of the company he worked for.

Part A

Mario has $759.60 in the bank after two years. Assuming he made no additional deposits or withdrawals, what simple interest rate did the savings account pay? Show or explain all your work.

Part B

Luis's stock is worth 5% more than Mario's savings after two years. By what percentage did the value of Luis's stock increase? Show or explain how you found your answer.